

Spatio-temporal models for Arctic seal abundance: successes and computational challenges

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March 1, 2018



Outline

- Ice-associated seals

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- Seal aerial surveys

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- Abundance and distribution modeling

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Ice-associated seals

Photo: Mike Cameron



Bearded seal

Photo: Josh London



Ribbon seal

Photo: Mike Cameron



Ringed seal

Photo: Jay Ver Hoef



Spotted seal

Ice-associated seals



Bearded seal



Ribbon seal



Ringed seal



Spotted seal

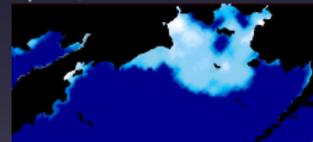
Seal aerial surveys

Joint U.S.-Russia Seal Surveys, 2012-2019

- Designed to provide spatially explicit, synoptic estimates of seal abundance over large areas (Okhotsk, Bering, Chukchi, and Beaufort Seas)
- Protocols that allow estimation of all forms of detection probability
- Analysis methods suitable for estimation under variable ice conditions



April 1, 2012

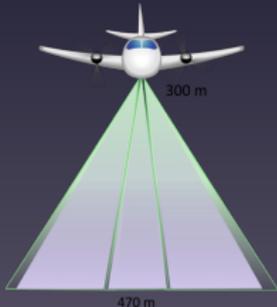


May 14, 2012

Joint U.S.-Russia Seal Surveys, 2012-2019

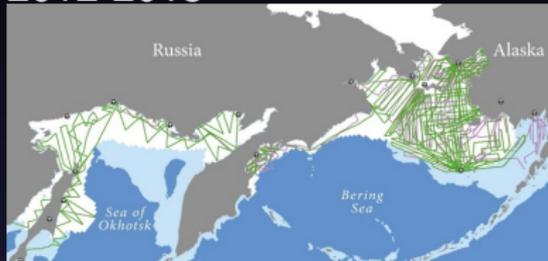
Instrument-based surveys

- Aerial surveys combining digital photography and thermal (infrared) video
- Model-based statistical inference: adjustment of flight tracks based on sea ice distribution



Joint U.S.-Russia Seal Surveys, 2012-2019

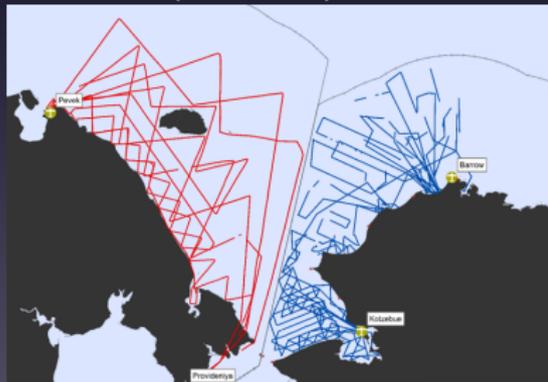
Bering-Okhotsk (BOSS) effort
2012-2013



CHESSE survey crew



Chukchi (CHESSE) effort 2016



Planned Beaufort effort, 2019



Abundance and distribution modeling

Components of detection



$$p_d p_a p_b$$



Eek!!

Abundance and distribution modeling

Components of detection: Species misclassification

Obs	Conf.	True Species			
		Bearded	Ribbon	Spotted	Ringed
Bearded	Certain	$\pi^{[1 1]}$	0	0	0
Bearded	Likely	$\pi^{[2 1]}$	$\pi^{[2 2]}$	$\pi^{[2 3]}$	$\pi^{[2 4]}$
Bearded	Guess	$\pi^{[3 1]}$	$\pi^{[3 2]}$	$\pi^{[3 3]}$	$\pi^{[3 4]}$
Ribbon	Certain	0	$\pi^{[4 2]}$	0	0
Ribbon	Likely	$\pi^{[5 1]}$	$\pi^{[5 2]}$	$\pi^{[5 3]}$	$\pi^{[5 4]}$
Ribbon	Guess	$\pi^{[6 1]}$	$\pi^{[6 2]}$	$\pi^{[6 3]}$	$\pi^{[6 4]}$
Ringed	Certain	0	0	$\pi^{[7 3]}$	0
Ringed	Likely	$\pi^{[8 1]}$	$\pi^{[8 2]}$	$\pi^{[8 3]}$	$\pi^{[8 4]}$
Ringed	Guess	$\pi^{[9 1]}$	$\pi^{[9 2]}$	$\pi^{[9 3]}$	$\pi^{[9 4]}$
Spotted	Certain	0	0	0	$\pi^{[10 4]}$
Spotted	Likely	$\pi^{[11 1]}$	$\pi^{[11 2]}$	$\pi^{[11 3]}$	$\pi^{[11 4]}$
Spotted	Guess	$\pi^{[12 1]}$	$\pi^{[12 2]}$	$\pi^{[12 3]}$	$\pi^{[12 4]}$
Unknown	NA	$\pi^{[13 1]}$	$\pi^{[13 2]}$	$\pi^{[13 3]}$	$\pi^{[13 4]}$

Abundance and distribution modeling

Components of detection: Species misclassification

Confusion Matrix - all observers combined (U.S.)

True Species	Observed Species				
	Bearded	Ribbon	Spotted	Ringed	Unknown
Bearded	0.83	0.04	0.05	0.04	0.04
Ribbon	0.00	0.98	0.00	0.01	0.00
Spotted	0.03	0.23	0.64	0.01	0.09
Ringed	0.01	0.04	0.07	0.86	0.02

B. T. McClintock, E. E. Moreland, J. M. London, S. P. Dahle, G. M. Brady, E. L. Richmond, K. M. Yano, and P. L.

Boveng. Quantitative assessment of species identification in aerial transect surveys for ice-associated seals.
Marine Mammal Science, 31:1057–1076, 2015

Abundance and distribution modeling

Data for spatio-temporal modeling

Discrete in space and time

Count data:

Unit (s)	Time (t)	$R_{s,t}$	$O_{s,t}^1$	$O_{s,t}^2$...	$O_{s,t}^{13}$
20	1	0.02	0	5	...	1
143	1	0.03	2	1	...	0
150	2	0.02	0	3	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮

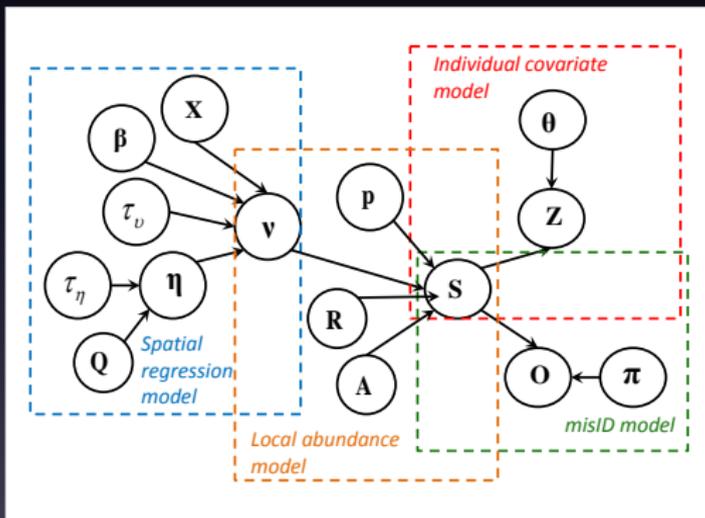
Environmental data:

Unit (s)	Time (t)	Size ($A_{s,t}$)	$X_{s,t}[1]$	$X_{s,t}[2]$...
1	1	1.0	0.5	3.4	...
2	1	0.9	0.3	2.1	...
3	1	1.0	0.8	0.1	...
⋮	⋮	⋮	⋮		

Abundance and distribution modeling

Initial analysis: spatial only, eastern Bering 2012

First step: restrict flights to a "short" window; ignore time



Methods in Ecology and Evolution

March 2013, Volume 4, Issue 3

doi: 10.1111/1365-3113.12127

SPECIAL ISSUE: MODELING DEMOGRAPHIC PROCESSES IN MARKED POPULATIONS: PROCEEDINGS OF THE EURING 2013 ANALYTICAL MEETING

Estimating multispecies abundance using automated detection systems: ice-associated seals in the Bering Sea

Paul S. Coen¹, Jay M. Ver Hoef, Brett T. McClintock, Eric S. Nordland, Josh M. London, Michael P. Cameron, Shawn P. Datta and Peter L. Bowler

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Summary

1. Automated detection systems employing advanced technology (e.g. satellite imagery, auditory recording systems, passive acoustic receivers) are expanding rapidly by providing spatial abundance and distribution data. However, many of these data are often difficult and expensive to collect and therefore reduce the capacity to survey using traditional techniques.

2. Even with these improvements, studying animal chemistry with advanced technology can be challenging because of potential for incomplete detection, false positives and species misidentification. We argue that double sampling with an independent sampling method can provide the critical information needed to account for such issues.

3. We present a hierarchical modeling framework for jointly using automated detection and double sampling data (instead of relying on traditional capture-recapture, capture-mark-recapture or different sampling methods) to estimate abundance in a known area (e.g. Bering Sea Chukchi Sea). Our process allows for spatial autocorrelation (using Bayesian methods), nonindependent observations, and complex, non-linear relationships between species characteristics (e.g. sex, season) and detection probability (e.g. sex, season, species characteristics).

4. As an example of modeling an ice-associated detection system and a double sampling population, we consider the problem of estimating annual abundance (over a 20-year time range) for three species, and their population growth rates. We illustrate the utility of double sampling in species composition and abundance estimation. We discuss our approach to handling unbalanced data and data from sampling of first-time-associated species in the eastern Bering Sea.

5. Our analysis and modeling performance of our hierarchical modeling approach, we suggest a need to balance model complexity with the volume of the data set. For example, highly parameterized models can lead to sparsely high prediction of abundance in areas that are not sampled, especially when data are sparse (e.g. spatial coverage).

6. We recommend that ecologists employ double sampling when estimating animal populations with automated detection systems to estimate and correct for detection errors. Combining multiple data sets within a hierarchical modeling framework provides a powerful approach for analyzing animal abundance over large spatial domains.

Keywords: abundance estimation, animal surveys, automated detection, data augmentation, hierarchical models, pattern recognition, spatially structured regression, species misidentification, thermal imaging

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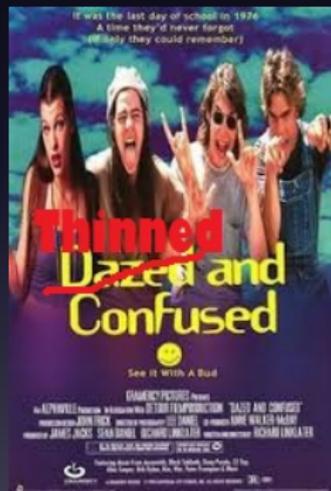
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Abundance and distribution modeling

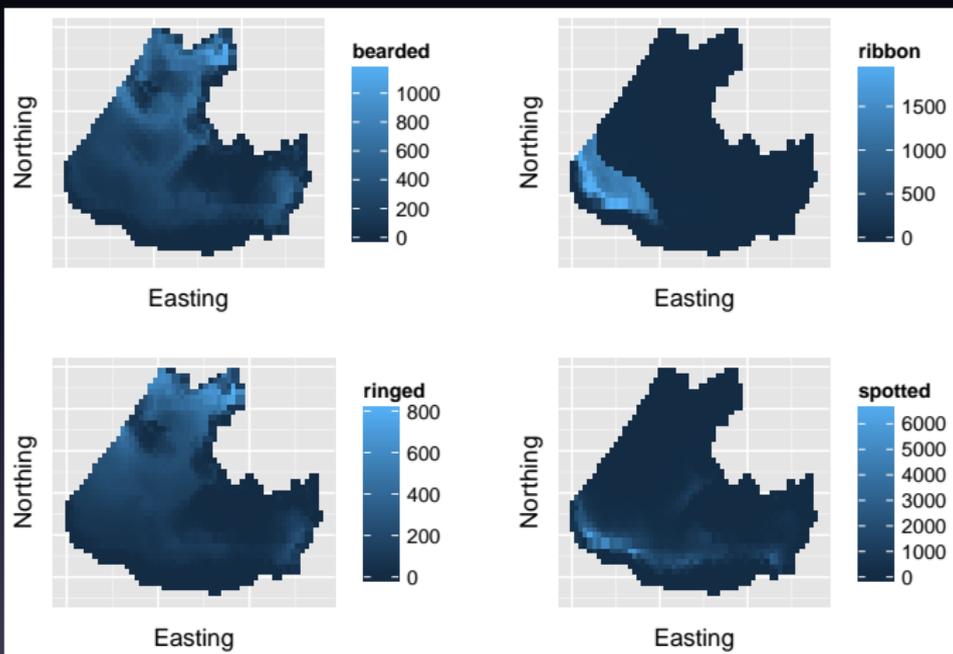
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First step: restrict flights to a “short” window; ignore time

→ “Thinned and confused”
log-Gaussian Cox process



Abundance and distribution modeling



Spatio-temporal seal analysis

Space-time: the final frontier!

Want to allow spatio-temporal autocorrelation in abundance.

Many options [Cressie and Wikle, 2011]!

- 1 Descriptive vs. dynamical

Spatio-temporal seal analysis

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- 2 Strategy for dimension reduction (knot based, spectral decomposition)

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- 3 Boundary conditions (in the case of dynamical S-T models)

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With regards to abundance estimation in particular:

- 1 Demographic closure

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With regards to abundance estimation in particular:

- 1 Demographic closure
- 2 Observation model and link function

P. B. Conn, D. S. Johnson, J. M. Ver Hoef, M. B. Hooten, J. M. London, and P. L. Boveng. Using spatio-temporal statistical models to estimate animal abundance and infer ecological dynamics from survey counts. Ecological Monographs, 85:235–252, 2015

Spatio-temporal models for abundance

Temporarily ignoring incomplete detection, how do several different forms of hierarchical spatio-temporal statistical models perform in estimating animal abundance from survey counts $(C_{s,t})$?

- 1 Descriptive additive space-time (AST) (*)

Spatio-temporal models for abundance

Temporarily ignoring incomplete detection, how do several different forms of hierarchical spatio-temporal statistical models perform in estimating animal abundance from survey counts $(C_{s,t})$?

- 1 Descriptive additive space-time (AST) (*)
- 2 Descriptive spatio-temporal process convolution (STPC) (*)

Spatio-temporal models for abundance

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- 1 Descriptive additive space-time (**AST**) (*)
- 2 Descriptive spatio-temporal process convolution (**STPC**) (*)
- 3 Dynamic open population resource selection (**OPRS**) (*)

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- 4 Descriptive multinomial closed population ideal free model (**CPIF**)

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- 3 Dynamic open population resource selection (**OPRS**) (*)
- 4 Descriptive multinomial closed population ideal free model (**CPIF**)

* based on Log-Gaussian Cox process

$$[\mathbf{C}] = \text{Poisson}(\boldsymbol{\lambda}),$$

$$\boldsymbol{\lambda} = \exp(\mathbf{o} + \mathbf{H}\boldsymbol{\mu})$$

$$N_{s,t} \sim \text{Poisson}(\exp(\mu)) \text{ [posterior prediction]}$$

AST model

Additive space-time (AST) model [similar to Ver Hoef and Jansen, 2007]:

$$\mu_{s,t} = \mathbf{X}_{s,t} \boldsymbol{\beta} + \eta_s + \gamma_t + \epsilon_{s,t}$$

$$\boldsymbol{\eta} = \mathbf{K} \boldsymbol{\alpha}$$

$$\boldsymbol{\alpha} \sim \mathcal{N}(0, \tau_{\alpha}^{-1})$$

$$\gamma_t \sim GMRF - RW2(\tau_{\gamma})$$

$$\epsilon_{s,t} \sim \mathcal{N}(0, \tau_{\epsilon}^{-1})$$

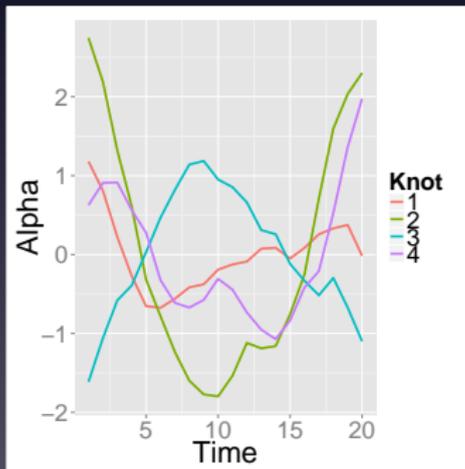
Here, \mathbf{K} is an $S \times m$ matrix with entries $K_{ij} = \mathcal{N}(d_{ij}; 0, \sigma^2)$ - where d_{ij} is the distance from the centroid of the i th sampling unit to the j th knot and σ is set a priori to the average distance between knots.

STPC model

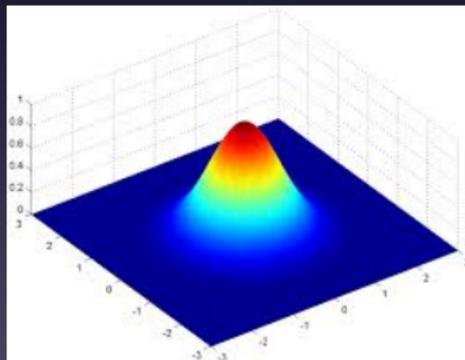
Spatio-temporal process convolution (STPC) model:

$$\begin{aligned}\boldsymbol{\mu} &= \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\kappa} + \boldsymbol{\epsilon} \\ \epsilon_{s,t} &\sim \mathcal{N}(0, \tau_{\epsilon}^{-1})\end{aligned}$$

Here, $\boldsymbol{\kappa}$ is spatio-temporal effect induced by dynamic process convolution [Calder et al., 2002]



+



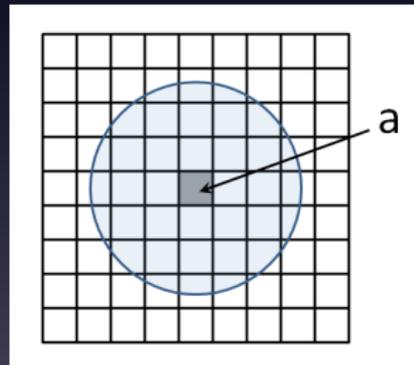
OPRS model

Open population resource selection (OPRS) model:

$$\mu_{s,t} = \begin{cases} \mathbf{X}_1\boldsymbol{\beta} + \boldsymbol{\eta} + \epsilon_1 & \text{if } t = 1 \\ \mathbf{M}_t\boldsymbol{\mu}_{t-1} + \boldsymbol{\gamma}_t + \boldsymbol{\epsilon}_t & \text{if } t > 1. \end{cases}$$

The elements of the transition matrix \mathbf{M}_t are determined via the “weighted distribution” typical in resource selection studies:

$$\begin{aligned} \psi_t^{ab} &= \frac{w_{b,t}\varphi_{a,b}}{\sum_s w_{s,t}\varphi_{a,s}} \\ \varphi_{a,b} &\propto \text{Normal}(d(a,b), \tau_d^{-1}), \\ \log(\mathbf{w}_t) &= \mathbf{X}_t\boldsymbol{\beta}. \end{aligned}$$



CPIF model

Closed population ideal free (CPIF) model

$$C_{s,t} \sim \text{binomial}(N_{s,t}, p_{st})$$

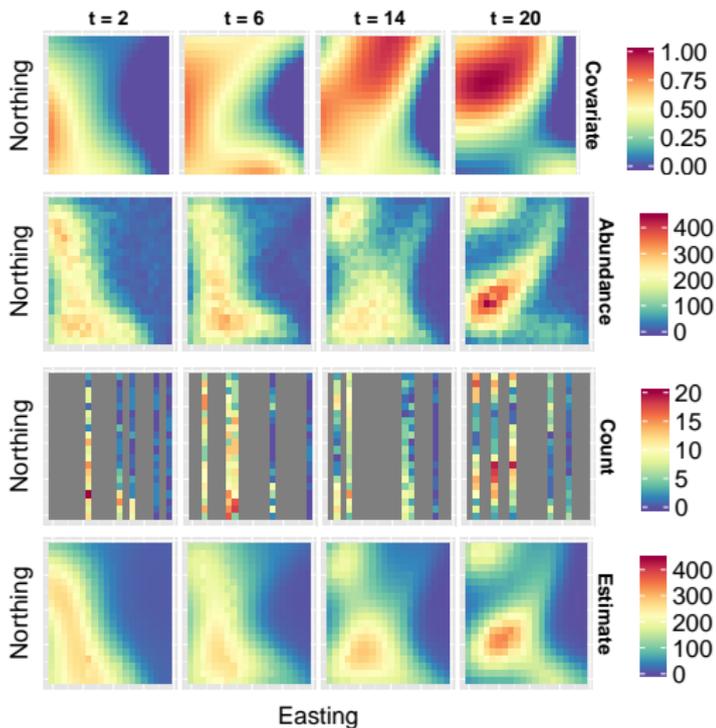
$$N_{s,t} \sim \text{multinomial}(N, \boldsymbol{\pi})$$

$$\pi_{i,t} = \frac{\exp(\omega_{i,t})}{\sum_s \exp(\omega_{s,t})}$$

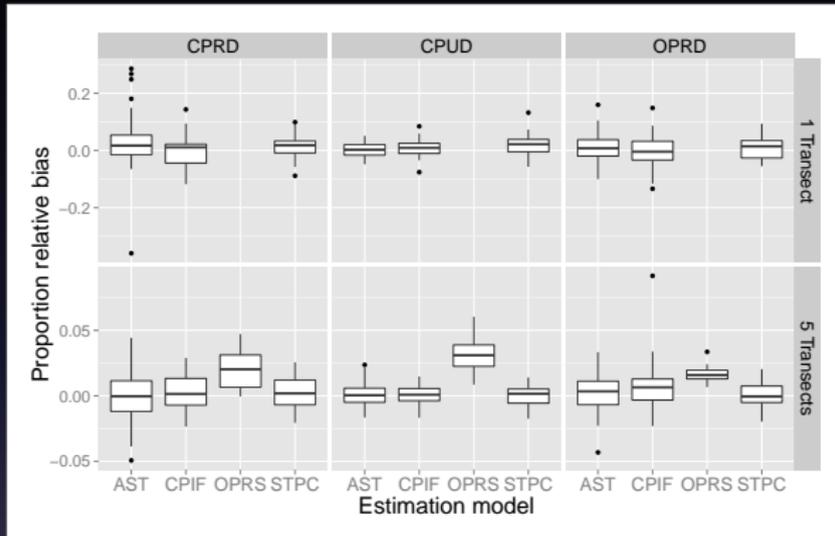
$$\boldsymbol{\omega}_t = \mathbf{0} + \mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\kappa}_t + \boldsymbol{\epsilon}_t$$

$$N \sim 1/N$$

Generic simulated data



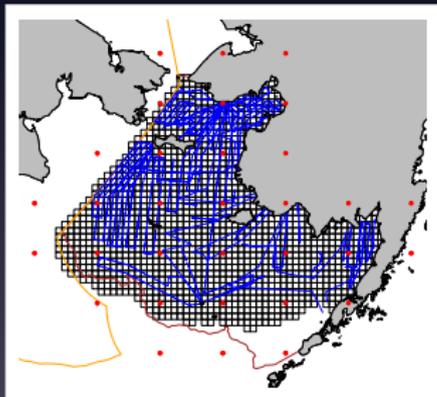
Generic simulated data



- Coverage often poor when different model used for estimation than for simulating data
- Computing hours/sim: ≈ 0.2 ; CPIF, STPC ≈ 1.5 ; OPRS 27-72
- Performance similar in a 2nd simulation study emulating seal surveys

Spatio-temporal seal analysis

We're currently working on applying the multinomial model to data from 2012 and 2013 Bering Sea aerial surveys, having added back in detection components (imperfect detection, availability < 1.0 , species misidentification).



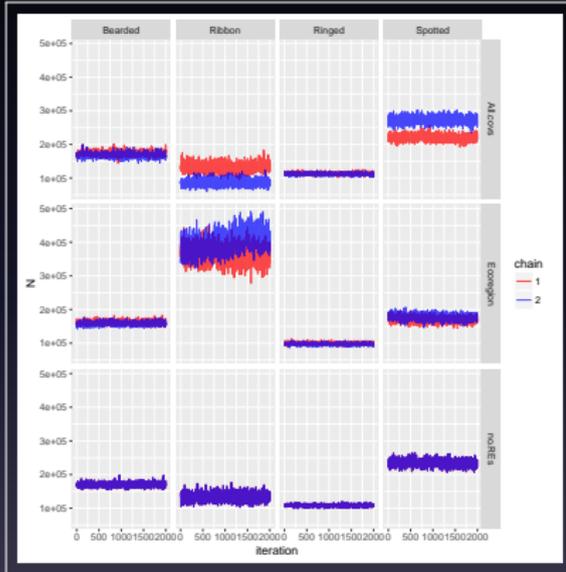
Predictive covariates

- Sea ice concentration (linear, quadratic)
- Distance from southern ice edge
- Distance from 1000m shelf break
- Distance from mainland
- Distance from a 90% sea ice contour

Spatio-temporal seal analysis

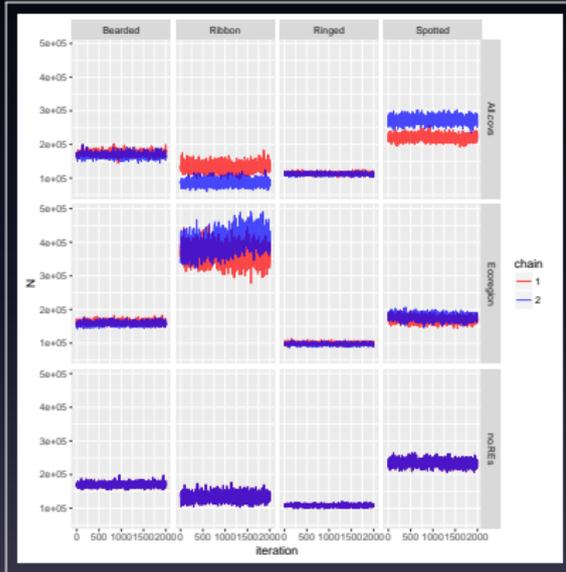
Videos!

Spatio-temporal seal analysis



- Large uncertainty about spotted, ribbon seal abundance in 2012
- Lack of convergence and evidence of multimodality in “complicated models”

Spatio-temporal seal analysis



Much better diagnostics in 2013.

- Large uncertainty about spotted, ribbon seal abundance in 2012
- Lack of convergence and evidence of multimodality in “complicated models”

Reasons to switch to TMB

- Bespoken MCMC sampler takes 7-10 days to run
- Lack of convergence
- Difficulty debugging and conducting adequate model testing
- It would be nice to implement new features quickly (e.g. pup recruitment processes, more flexibility in availability).

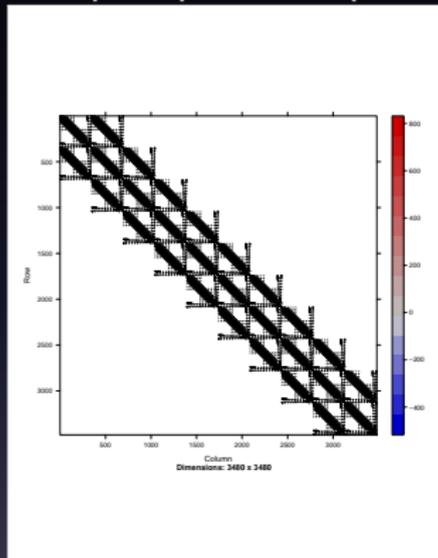
Implementation in TMB

Simple spatio-temporal Poisson model:

$$\begin{aligned}C_{stk} &\sim \text{Poisson}(p\tilde{\lambda}_{stk}) \\ \tilde{\lambda}_{st} &= \psi\lambda_{st} \\ \log(\lambda_{tk}) &= \mathbf{o} + \mathbf{X}_{tk}\beta_k + \eta_{tk} \\ \eta_k &\sim \text{MVN}(\mathbf{0}, \Sigma_{AR1,k} \otimes \Sigma_{GMRF,k}) \\ p &\sim [p] \\ \psi &\sim [\psi]\end{aligned}$$

Implementation in TMB

Simple spatio-temporal Poisson model:



- 3 species, 1600 cells, 10 time steps: >48,000 REs. Okay!
- 3 species, 2500 cells, 10 time steps: >75,000 REs. Error in `sparseHessianFun(env, skipFixedEffects = skipFixedEffects)`: Memory allocation fail in function 'MakeADHessObject2'

Possible fix: knot-based interpolation such as piece-wise constant (VAST) or predictive process [Banerjee et al., 2008], basis function decomposition

Implementation in TMB

CPIF-like species redistribution model using Poisson approx to multinomial and separable covariance:

$$C_{stk} \sim \text{Poisson}(p\tilde{\lambda}_{stk})$$

$$\tilde{\lambda}_{st} = \psi\lambda_{st}$$

$$\lambda_{stk} = N_k\pi_{stk}$$

$$\pi_{stk} = \frac{\exp(\omega_{stk})}{\sum_s \exp(\omega_{stk})}$$

$$\omega_{tk} = \mathbf{0} + \mathbf{X}_{tk}\beta_k + \eta_{tk}$$

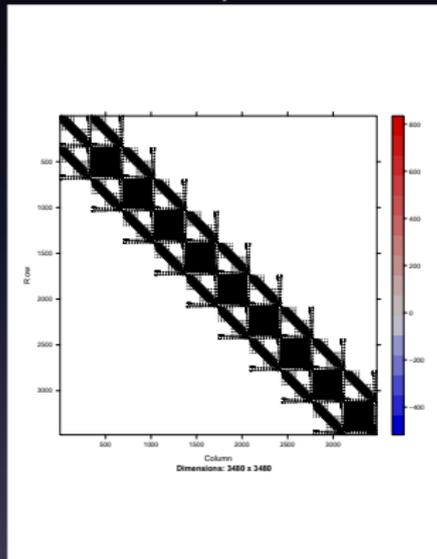
$$\eta_k \sim \text{MVN}(\mathbf{0}, \Sigma_{AR1,k} \otimes \Sigma_{GMRF,k})$$

$$p \sim [p]$$

$$\psi \sim [\psi]$$

Implementation in TMB

CPIF-like species redistribution model



25×25 grid, 3 species, $> 18750REs$:
Runs but slow

30×30 grid, 10 time steps, 3 species,
 $> 27000REs$: Memory allocation fail
→ Need to use some form of
dimension reduction.

Summary

There are many spatio-temporal modeling options

- 1 Choice of link function
- 2 Basis choice
- 3 Descriptive vs. dynamical

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And software options!

- 1 Simplest: Canned packages using GLMM-like syntax (mgcv, INLA)
- 2 More complex: Nonlinear and “non-standard” models in TMB
- 3 Most complex: Fully Bayesian: multiple levels of hierarchy, discrete latent variables, dynamical models

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There are many spatio-temporal modeling options

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- 3 Most complex: Fully Bayesian: multiple levels of hierarchy, discrete latent variables, dynamical models

There are a lot of tradeoffs! (Ease of implementation, flexibility, realism)

Acknowledgments

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- Many coauthors and partners on parts of this work, including: E. Moreland, M. Cameron, J. Ver Hoef, J. London, M. Hooten, D. Johnson, E. Richmond, S. Dahle, B. McClintock, P. Boveng, I. Trukhovana
- All researchers and technicians who assisted with Arctic seal surveys



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